# Using Like Bases to Solve Exponential Equations

For any algebraic expression and , and any positive real number ,

if and only if

In other words, when an exponential equation has the same base on each side, the exponents must be equal.

Examples

1)

2)

3)

4)

5)

6)

# Solving Exponential Equations Using Logarithms

Sometimes the terms of an exponential equation cannot be rewritten with a common base. In these cases, we solve by taking the logarithm of each side. Recall, since  is equivalent to , we may apply logarithms with the same base on both sides of an exponential equation.

Given an exponential equation in which a common base cannot be found, solve for the unknown by

Applying the logarithm to both sides of the equation.

If one of the terms in the equation has base 10, use the common logarithm.

If none of the terms in the equation has base 10, use the natural logarithm.

Use the rules of logarithms to solve for the unknown.

Examples: Solve each of the following equations using logarithms.

# Using the Definition of a Logarithm to Solve Logarithmic Equations

Remember, you can write a logarithmic equation as an exponential equation and vice versa. This is especially helpful if you have an equation with a single logarithm. Also, be sure to check your answer in the original equation and reject any value that generates a negative argument!

For any algebraic expression and real numbers and , where ,

if and only if

Examples

1)

2)

3)

4)

5)

# Using the One-to-One Property of Logarithms to Solve Logarithmic Equations

For any algebraic expressions and and any positive real number , where ,

if and only if

\*Note, when solving an equation involving logarithms, always check to see if the answer is correct or if it is an extraneous solution.

Given an equation containing logarithms, we can solve it using the one-to-one property by

Using the rules of logarithms to combine like terms, if necessary, so that the resulting equation has the form .

Using the one-to-one property to set the arguments equal.

Solving the resulting equation, , for unknown.

Examples: Solve the following logarithmic equations using the one-to-one property.

# Solving Applied Problems Using Exponential and Logarithmic Equations

In previous sections, we learned the properties and rules for both exponential and logarithmic functions. We have seen that any exponential function can be written as a logarithmic function and vice versa. We have used exponents to solve logarithmic equations and logarithms to solve exponential equations. We are now ready to combine our skills to solve equations that model real-world situations, whether the unknown is in an exponent or in the argument of a logarithm.

Examples:

1. An account with an initial deposit of earns annual interest, compounded continuously. How much will the account be worth after 20 years?
2. The formula for measuring sound intensity in decibels is defined by the equation , where is the intensity of the sound in watts per square meter and is the lowest level of sound that the average person can hear. How many decibels are emitted from a jet plane with a sound intensity of watts per square meter?
3. The population of a small town is modeled by the equation where is measured in years. In approximately how many years will the town’s population reach ?
4. How long will it take for ten percent of a 1000-gram sample of uranium-235 to decay?